

# Engineering of oscillatory quantum states by parametric excitation

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We consider preparation of nonclassical oscillatory states in a degenerate parametric oscillator combined with phase modulation. In this scheme intracavity oscillatory mode is excited by train of Gaussian laser pulses through degenerate down-conversion process and phase modulation element inserted in a cavity leads to anharmonicity of oscillatory mode. We demonstrate production of nonclassical oscillatory states with two-fold symmetry in phase-space including the superposition of Fock states and quantum localized states on the level of few excitation numbers and in over-transient dissipative regime.

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## I. INTRODUCTION

There is currently a wide effort to construct various artificial nonlinear oscillatory systems showing quantum behavior. Such systems operated in quantum regime have become more important in both fundamental and applied sciences, particularly, for implementation of basic quantum optical systems, in engineering of nonclassical states and quantum logic. In these systems the efficiency of quantum effects requires a high nonlinearity with respect to dissipation. The nonlinearity breaks the equidistance of oscillatory energy levels that allows selectively excites the oscillatory states by monochromatic driving analogous to that take place for electronic states of atomic systems. Thus, the limited two-level and three-level systems can be realized approximately for oscillatory systems.

An oscillator operated in quantum regime is naturally described by Fock states that are states with definite numbers of energy quanta. The preparation and use of Fock states and various superpositions of Fock states form the basis of quantum computation and communications [1]. However, excitations of oscillatory systems usually lead to production of coherent states nearly indistinguishable from a classical state, but not quantum Fock states. In this reason quantum oscillatory states are usually prepared and manipulated by coupling oscillators to atomic systems. In this way, a classical pulse applied to the atomic states creates a quantum state that can subsequently be transferred to the harmonic oscillator excited in a coherent state. The systematic procedure has been proposed in [2] and has been demonstrated for deterministic preparation of mechanical oscillatory Fock states with trapped ions [3] and in cavity QED with Rydberg atoms [4]. Most recently the analogous procedure has been applied in solid-state circuit QED for deterministic preparation of photon number states in a resonator

by interposing a highly nonlinear Josephson phase qubit between a superconductive resonator [5].

Recently, it has been shown that production of Fock states and Fock states superpositions or qubits can also be realized in over-transient regime of an anharmonic dissipative oscillator without any interactions with atomic and spin-1/2 systems [6]. Preparation of the lower Fock states  $|1\rangle$  and superposition state  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$  has been demonstrated for the lowest excitation  $|0\rangle \rightarrow |1\rangle$  with complete consideration of decoherence effects. For this goal the strong Kerr nonlinearity as well as the excitation of resolved lower oscillatory energy levels with specific train of Gaussian pulses have been considered.

In continuation of this paper [6] here we propose the other oscillatory scheme based on two-quantum resolved excitations of oscillatory levels  $|n\rangle \rightarrow |n+2\rangle$ . Thus, this scheme is the Duffing oscillator for the mode created in the process of degenerate down-conversion under an external field. One of the possible realizations of this scheme is the parametrically driven anharmonic oscillator (PDAO) that consists from intracavity degenerate parametric oscillator combined with phase modulation element inserted in a cavity. In this scheme intracavity oscillatory mode is excited by strong field through degenerate down-conversion and phase modulation leads to anharmonicity of oscillatory mode.

Note, that parametrically driven anharmonic oscillator based on cascaded parametric oscillator and third-order phase modulation has been proposed and studied in the papers [7], [8] for the special cases without consideration of effects of dissipation and quantum fluctuations. Quantum theory of monochromatically driven parametric oscillator combined by phase modulation has been developed in terms of the Fokker-Planck equation in complex P representation [9], [10]. The exactly solution of this equation (that means consideration of all order of dissipation) has been obtained in steady-state regime and the Wigner function of oscillatory mode has been obtained in analytical form using this solution [9]. These results strongly demonstrate the vanishing of Fock states and quantum superposition states in over transient dissipa-

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tive operational regime of PDAO within framework of the Wigner function that visualizes quantum effects as negative values in phase-space. Really, analytical results for the Wigner functions of PDAO under monochromatic excitation [9] has been calculated as positive in all ranges of phase space in the steady-state regime.

In this paper, we demonstrate that in the specific pulsed regime of PDAO the production of nonclassical oscillatory states with two-fold symmetry in phase-space consisting from the Fock states  $|0\rangle$ ,  $|2\rangle$ , as well as superposition of Fock states  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$  are realized. These states can be created for time intervals exceeding the characteristic time of decoherence.

The quantum regime of PDAO requires a high third-order, Kerr nonlinearity with respect to dissipation. The largest Kerr nonlinearities for oscillatory systems were proposed for cooling nano-electromechanical systems and nano-opto-mechanical systems based on various oscillators [14],[15]. Superconducting devices based on the nonlinearity of the Josephson junction (JJ) that exhibits a wide variety quantum phenomena [16]-[22] offer an unprecedented high level of nonlinearity and low quantum noise. In some of these devices dynamics are analogous to those of a quantum particle in an oscillatory anharmonic potential [23], [24]. Note, that comparison of third-order nonlinearities taking place for various quantum devices have been recently analyzed in [25].

The paper is arranged as follows. In Sec. II we describe parametrically driven nonlinear oscillator under pulsed excitation and describe phase-space symmetry properties of the model. In Sec. III we shortly discuss PDAO under a monochromatic driving for both transient and steady-state regimes. In Sec. IV we consider production of nonclassical oscillatory states in the pulsed regime of PDAO. We summarize our results in Sec. V.

## II. MODEL DESCRIPTION

In this section we give the theoretical description of the system. The nonlinear oscillator driven parametrically by train of pulses and interacting with a reservoir is described by the following Hamiltonian

$$H = \hbar\omega_0 a^\dagger a + \hbar\chi(a^\dagger a)^2 + \hbar\Omega(E(t)e^{-i\omega t}a^{+2} + E^+(t)e^{i\omega t}a^2) + H_{loss}, \quad (1)$$

where  $a^\dagger$ ,  $a$  are the oscillatory creation and annihilation operators,  $\omega_0$  is the oscillatory frequency,  $\chi$  is the nonlinearity strength proportional to the third-order susceptibility. The coupling constant  $\Omega$  is proportional to the second-order susceptibility and the time-dependent amplitude of the driving field  $E(t) = E_0 f(t)$  consists from the Gaussian pulses with the duration  $T$  which are separated by time intervals  $\tau$

$$f(t) = \sum e^{-(t-t_0-n\tau)^2/T^2}. \quad (2)$$

$H_{loss} = a\Gamma^\dagger + a^\dagger\Gamma$  is responsible for the linear losses of oscillatory state, due to couplings with heat reservoir operators giving rise to the damping rate  $\gamma$ . The reduced density operator  $\rho$  within the framework of the rotating-wave approximation, in the interaction picture corresponding to the transformation  $\rho \rightarrow e^{-i(\omega/2)a^\dagger a t} \rho e^{i(\omega/2)a^\dagger a t}$  is governed by the master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_0 + H_{int}, \rho] + \sum_{i=1,2} \left( L_i \rho L_i^\dagger - \frac{1}{2} L_i^\dagger L_i \rho - \frac{1}{2} \rho L_i^\dagger L_i \right), \quad (3)$$

where  $L_1 = \sqrt{(N+1)\gamma}a$  and  $L_2 = \sqrt{N\gamma}a^\dagger$  are the Lindblad operators,  $\gamma$  is a dissipation rate and  $N$  denotes the mean number of quanta of heat bath,

$$H_0 = \hbar\Delta a^\dagger a, \\ H_{int} = \hbar\chi(a^\dagger a)^2 + \hbar\Omega(E(t)a^{+2} + E(t)^*a^2), \quad (4)$$

and  $\Delta = \omega_0 - \omega/2$  is the detuning between half frequency of the driving field  $\omega/2$  and the oscillatory frequency  $\omega_0$ . Two last terms in the interaction Hamiltonian describe the self-phase modulation (SPM) of the oscillatory mode and the parametric three-wave interaction between semiclassical driving field and the oscillatory mode, respectively.

To study the pure quantum effects we focus on the cases of very low reservoir temperatures which, however, ought to be still larger than the characteristic temperature  $T \gg T_{cr} = \hbar\gamma/k_B$ . This restriction implies that dissipative effects can be described self-consistently in the frame of the Linblad Eq. (3). For clarity, in our numerical calculation we choose the mean number of reservoir photons  $N = 0$ .

It is evident that the system displays definite symmetry properties in phase-space. Really, considering the transformations

$$H' = U^{-1} H U, \quad \rho' = U^{-1} \rho U \quad (5)$$

with the unitary operator

$$U = \exp(i\theta a^\dagger a). \quad (6)$$

we verify that the interaction Hamiltonian (4) satisfies the commutation relation

$$[H, U] = 0. \quad (7)$$

The analogous symmetry takes place for the density operator of oscillatory mode

$$[\rho(t), U] = 0. \quad (8)$$

One of the most important conclusions of such symmetries is related to the Wigner functions of the oscillatory mode

$$W(\alpha) = \frac{1}{\pi^2} \int d^2\gamma \text{Tr} \left( \rho e^{\gamma a^\dagger - \gamma^* a} \right) e^{\gamma^* \alpha - \gamma \alpha^*}, \quad (9)$$

where we perform rotations by the angle  $\theta$  around the origin in phase spaces of complex variables  $\alpha$  corresponding to the field operators  $a$  in the positive P-representation. Indeed, in the polar coordinates  $r, \theta$  of the complex phase-space plane  $X = (\alpha + \alpha^*)/2 = r \cos \theta$ ,  $Y = (\alpha - \alpha^*)/2i = r \sin \theta$  we derive that the Wigner function displays two-fold symmetry in its rotation around the origin of phase-space

$$W(r, \theta + \pi) = W(r, \theta). \quad (10)$$

It is well known that this symmetry take place for the degenerate optical parametric oscillator (OPO) and reflects on the phase-locking phenomenon in above threshold regime of OPO. According to phase-locking the mode of sub-harmonic generated in OPO are produced with well-defined two phases [27]. The result (10) shows that this situation takes place also for the combined system under consideration. The illustrations will be presented below on the Figs. 4, 6.

In the following the distribution of oscillatory excitation states  $P(n) = \langle n|\rho|n \rangle$  as well as the Wigner functions

$$W(r, \theta) = \sum_{n,m} \rho_{nm}(t) W_{mn}(r, \theta) \quad (11)$$

in terms of the matrix elements  $\rho_{nm} = \langle n|\rho|m \rangle$  of the density operator in the Fock state representation will be calculated. Here the coefficients  $W_{mn}(r, \theta)$  are the Fourier transform of matrix elements of the Wigner characteristic function.

In this paper, we demonstrate that in the specific pulsed regime of PDAO and for the case of strong nonlinearities the Fock states as well as superposition of Fock states are realized. These states can be created for time intervals exceeding the characteristic time of decoherence. The corresponding Wigner functions of oscillatory mode show ranges of negative values and gradually deviate from the Wigner function of an PDAO driven by monochromatic driving.

Nevertheless, the time evolution of PDAO driven by a coherent force cannot be solved analytically for arbitrary evolution times and suitable numerical methods have to be used. We solve the master equation Eq. (3) numerically based on quantum state diffusion method [28]. The applications of this method for studies of the driven nonlinear oscillators and the parametric optical oscillator can be found in [29]-[31]. In the calculations a finite bases of number states  $|n\rangle$  is kept large enough (with  $n_{max}$  is typically 50) so that the highest energy states are never populated appreciably.

It should be also noted that the investigation of quantum dynamics of a driven dissipative nonlinear oscillator

for nonstationary cases, particularly, for various pulsed regimes, is much more complicated and only a few papers have been done in this field up to now. Quantum effects in nonlinear dissipative oscillator with time-modulated driving force have been studied in the series of the papers [29]-[32] in the context of a quantum stochastic resonance [29], quantum dissipative chaos [30], [31], and quantum interference assisted by a bistability [32].

### III. PDAO IN MONOCHROMATIC FIELD

In this section, we shortly discuss the combined dissipative oscillator under monochromatic excitation, considering  $f(t) = 1$  in the Hamiltonian (4). At first, we present the results in the semiclassical approach and the standard linear stability analysis with respect to small deviations from steady state in terms of the amplitude of the oscillatory mode  $\alpha = n^{1/2} \exp(i\varphi)$  [9]. The intensity  $n$  (in photon number units) and the phase of the mode  $\varphi$  in stable above-threshold regime is determined by the following expressions:

$$n = \frac{\gamma}{2\chi} \left[ \frac{\Delta}{\gamma} + (J - 1)^{1/2} \right], \quad \sin(\Phi - 2\varphi) = J^{-1/2}, \quad (12)$$

where  $J = \frac{\Omega^2}{\gamma^2} I$  and  $\Phi$  is the phase of the driving field  $E_0 = I^{1/2} \exp(i\Phi)$ . Above threshold regime takes place for  $I > I_{th} = \frac{\gamma^2}{\Omega^2} (1 + \frac{\Delta^2}{\gamma^2})$  and the regular behaviour is realized for negative detunings. These results are obtained in over transient regime and for large oscillatory mean excitation numbers  $n \gg 1$ .

We now present results for strong quantum regime of PDNO that is realized for  $\chi/\gamma \geq 1$ . In the absence of any driving, the quantized vibration states of nonlinear oscillator are the Fock states  $|n\rangle$  which are spaced in energy  $E_n = E_0 + \hbar\omega_0 n + \hbar\chi n^2$  with  $n = 0, 1, \dots$ . The levels form an anharmonic ladder (see, Fig. 1(a)) with anharmonicity that is given by  $E_{21} - E_{10} = 2\hbar\chi$ . Below we concentrate on quantum regimes for the parameters when oscillatory energy levels are well resolved considering near to resonant two-photon transitions between lower number states  $|0\rangle \rightarrow |2\rangle$ ,  $|1\rangle \rightarrow |3\rangle$ . Thus, we assume non strong excitation regime  $E\Omega/\gamma = 7$  and that the detuning  $\delta_2 = \frac{1}{\hbar}(E_2 - E_0) - \omega = 2\Delta + 4\chi$  meets the near to resonant condition,  $\delta_2 = 16\gamma$ . The detuning between the transition  $|1\rangle \rightarrow |3\rangle$  reads as  $\delta_3 = \frac{1}{\hbar}(E_3 - E_1) - \omega = 2\Delta + 8\chi$  and for this parameters is more large than  $\delta_2$ , i.e.  $\delta_3 = 36\gamma$ . For comparison we add in Fig. 1(b) also the lower levels and the oscillatory transitions for the scheme considered in [6].

Now, we present on Fig. 2 the numerical results for monochromatically driven PDAO in dependence of the parameters:  $\chi, \Delta, E\Omega$ . Transition the system to the steady state for time intervals  $t \gg \gamma^{-1}$  is depicted in Figs. 2 (a,b) on the time-dependent mean excitation number

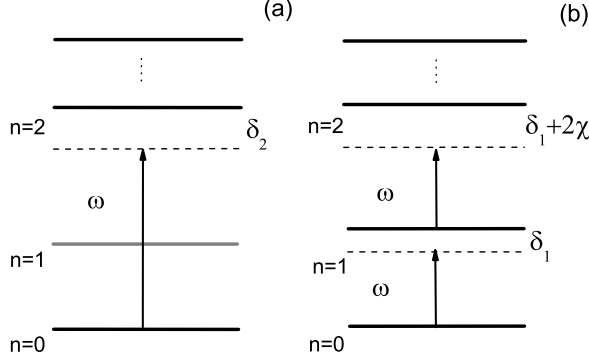


FIG. 1. Transitions between energetic levels of anharmonic oscillator: parametric driving with two-quanta detunings  $\Delta = \omega_0 - \omega/2$  and  $\delta_2 = \frac{1}{\hbar}(E_2 - E_0) - \omega = 2\Delta + 4\chi$  (a); standard driving with one-quanta detunings  $\Delta = \omega_0 - \omega$  and  $\delta_1 = \frac{1}{\hbar}(E_1 - E_0) - \omega = \Delta + \chi$  (b).

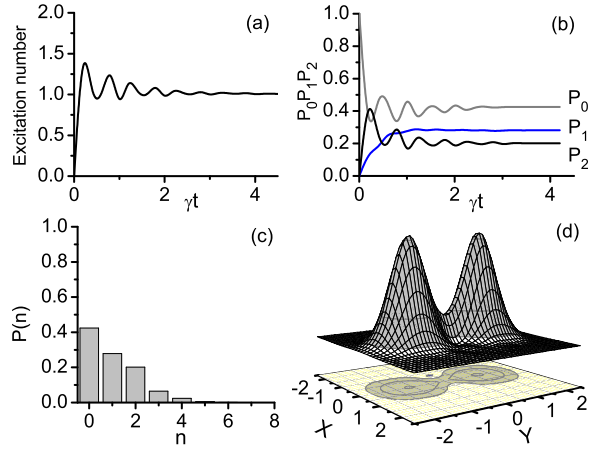


FIG. 2. Time evolution of the averaged excitation numbers (a); the Rabi oscillations of the state populations with decoherence which suppresses beating (b); the distribution of excitation numbers (c); and the Wigner function (d) for PDAO in steady-state regime. The parameters are:  $\Delta/\gamma = -2$ ,  $\chi/\gamma = 5$ ,  $E\Omega/\gamma = 7$

and the populations of Fock states. As we noted above, steady state results, in that number probability distribution of oscillatory excitation numbers and the Wigner function of oscillatory mode have been obtained analytically in terms of the exact solution of the Fokker-Planck equation [9], [10]. Particularly, the solution for the Wigner function of oscillatory mode involving quantum noise in all order of perturbation theory and in steady-state regime is positive in all phase space and hence does not describe Fock states in over-transient regime. Note,

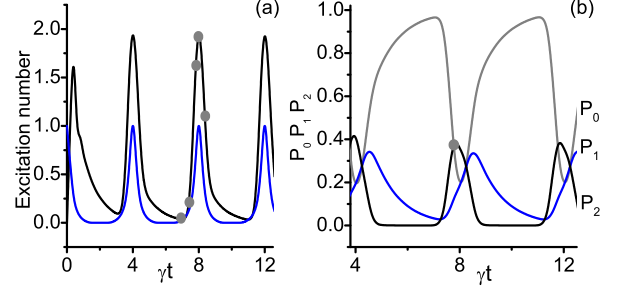


FIG. 3. Time-evolution of averaged excitation number (a) and populations of Fock states (b). Time-dependent structure of pulses is indicated below  $n(t)$  in arbitrary units. The parameters are:  $\Delta/\gamma = -2$ ,  $\chi/\gamma = 5$ ,  $E\Omega/\gamma = 10$ ,  $\tau = 4\gamma^{-1}$ ,  $T = 0.5\gamma^{-1}$

that the steady-state solution of the Fokker-Planck equation has been found using the approximation method of potential equations [33]. The validity of this solution has not been checked in the strong quantum regime of PDAO operated on the level of few excitation number. On this reason we calculate the Wigner function numerically on the base of numerical simulation of master equation by using quantum state diffusion method [28]. The results are displayed in the Figs.2 (d). As we see, the Wigner function displays two humps and has two-fold symmetry in phase-space under the rotation on angle  $\pi$  around its origin (10). This effect reflects the well-known phenomena of phase locking that takes place for both OPO and OPO combined with phase modulation. In semiclassical approach according to the equation (12) phase-locking in above threshold regime means the forming of two stable states of oscillatory mode with equal photon numbers, but with two different phases which are  $\Phi/2$  and  $\Phi/2 + \pi$ . In quantum treatment of PDAO this phenomenon is displayed as two-humped structure of the Wigner function as is has been demonstrated analytically in [10]. Here we demonstrate that similar effect occurs also for PDAO in strong quantum regime on the level of few excitation numbers.

#### IV. PRODUCTION OF OSCILLATORY QUANTUM STATES IN THE PULSED REGIME

Now we are able to present the results concerning production of oscillatory quantum states for pulsed excitation regime of PDAO. Since in this paper we focus on a system that remains close to its ground, vacuum state, we assume a small number of excitation number in the oscillator. We consider interaction time intervals exceeding the characteristic time of dissipative processes,  $t \gg \gamma^{-1}$ , however, in the nonstationary regime that is conditioned by the specific form of pulsed excitation. If the parametric excitation stipulated by train of the Gaussian pulses

the ensemble-averaged mean oscillatory excitation numbers, the populations of oscillatory states and the Wigner functions are nonstationary and exhibit a periodic time dependent behavior, i.e. repeat the periodicity of the laser pulses in an over transient regime.

As it has been shown in [6] the most striking signature of periodically pulsed NDO is the appearance of various quantum states involving Fock states and interference between Fock states in over transient regime. Excitation due to parametric interaction leads to production of new class of oscillatory states that have not been possible to generate in standard way by one-photon excitation. These states include squeezed states or superpositions of Fock state, as well as quantum localized states on the level of few excitation numbers. We demonstrate this point for near to two-quanta resonant transitions  $|n\rangle \rightarrow |n+2\rangle$  for the oscillatory parameters used above in Figs. 3 and 4 however, for the periodically pulsed regime with the duration  $T = 0.5\gamma^{-1}$ , and the time interval between pulses  $\tau = 4\gamma^{-1}$ . The evolution of the averaged excitation numbers and the population of the lowest states  $|0\rangle$ ,  $|1\rangle$  and  $|2\rangle$  are depicted in Figs. 3 while time-dependent structure of pulses is indicated below  $n(t)$  in arbitrary units. As we see, for over transient regime the time-modulation of the averaged excitation number and the populations of oscillatory states repeat the periodicity of the pump laser. Besides these results, the excitation numbers for the definite time-intervals during pulses and the corresponding Wigner functions of oscillatory mode are shown in Figs. 4. As we see the evolution of anharmonic oscillator under parametric excitation involves number of quantum oscillatory states which are visualized on the Wigner functions for typical five measurement time-intervals. We indicate the choosing of these time-intervals on the curve of main excitation number (see, Fig.3 (a)). For  $t = k\tau - 2T$ ,  $k = 2, 3, \dots$  the oscillatory mode is in the vacuum state, therefore the corresponding Wigner function is Gaussian (see, Fig.4 (f)). After evolution of the system, for the time intervals  $t = k\tau - 1.8T$ , the state is squeezed in phase space. The typical result characterizing parametric double excitation of oscillatory mode is effective simultaneous production of  $|0\rangle$  and  $|2\rangle$  states that are depicted in Fig. 4 (c) for the time intervals  $t = k\tau - 0.4T$ . The populations of these states is approximately equal one to the other and the corresponding Wigner function (see, Fig.4 (h)) demonstrates the interference fringes on phase-space between  $|0\rangle$  and  $|2\rangle$  states. It is easy understand that this Wigner function approximately describes the pure superposition state  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$ , i.e. for these time-interval PDAO produces superposition of Fock states  $|0\rangle$  and  $|2\rangle$ . It is important that this quantum interference effect is realized when the oscillatory mean excitation number reaches its maximal value. For further increasing of time interval, i.e. for  $t = k\tau$ , the interference fringes on phase-space are deformed and the Wigner function consists from two localized peaks with the ranges of negativity between them (see, Fig.4 (d, i)). Then, near to the end of the pulses,

i.e.  $t = k\tau + 0.6T$ , the quantum interference is vanished and production of localized state takes place for PDAO (see, Fig.4 (e,f)) as has been obtained in the steady state regime. It should be noted that these results are in accordance with the phase symmetry properties of PDAO (10). We conclude that production of quantum interference is realized in the vicinity of time-intervals where populations of  $|0\rangle$  and  $|2\rangle$  states are crossed (see, Fig 3(b)) in over transient regime. We assume that the control of decoherence in this case take place due to application of suitable tailored, synchronized pulses (see, for example, [34], [35]). Indeed, quantum interference is realized here if a mutual influence of pulses is essential, (for  $\tau/T = 3.14$  on Fig. 4).

The next regime that we consider is that where the pulsed excitation is tuned to the exact resonance,  $\delta = 2\Delta + 4\chi = 0$ . It is evident that in this case the Fock state  $|2\rangle$  can be effectively produced if a low excitation is used. Similarly to what was done above in dispersive regime, we consider at first the evolution of averaged excitation numbers as well as the population of the lowest states  $|0\rangle$ ,  $|1\rangle$  and  $|2\rangle$  by two-quanta excitation. The results depicted in Figs. 5 allows us to choose the time-intervals within a pulse for which the maximal probability of production of  $|2\rangle$  Fock state is realized. The time evolution of the probabilities  $P_0$ ,  $P_1$  and  $P_2$  of  $|0\rangle$ ,  $|1\rangle$  and  $|2\rangle$  states starting from the vacuum state are depicted in Fig. 5(b). As we see, that the maximal weight 0.6 of  $P_2$  is realized for the definite time-intervals of measurement  $t = k\tau - 0.25T$ ,  $k = 1, 2, 3, \dots$  (see, Fig. 5 (a)). In this way, for the duration of pulses  $T = 0.5\gamma^{-1}$ , and the time interval between them  $\tau = 4\gamma^{-1}$  the Wigner function is depicted in Fig. 6(b) for time measurement intervals  $t = k\tau + 0.8T$ . One might expect that this result would be approximately close to the pure  $|2\rangle$  Fock state Wigner function displayed for comparison in Fig. 6(a). This Wigner function displays ring signature with the center at  $x = y = 0$  in phase-space. Indeed, in the general these results are qualitatively similar involving also negative part, however, the cyclic symmetry of pure  $|2\rangle$  has not displayed in state of PDAO that acquires two-fold symmetry in phase-space due to parametric excitation.

## V. CONCLUSION

We have demonstrated production of various quantum states for the parametrically driven anharmonic oscillator in the regimes of low excitation and in complete consideration of dissipative effects. This investigation continues our previous analysis [6] devoted to creation of Fock states as well as superpositions of Fock states in the specific regime of periodically pulsed anharmonic oscillator for time-intervals exceeding the characteristic decoherence time. In this paper, we have proposed the other nonlinear oscillatory system that is excited by the train of Gaussian laser pulses through the degenerate down-conversion process. It can be realized as OPO combined

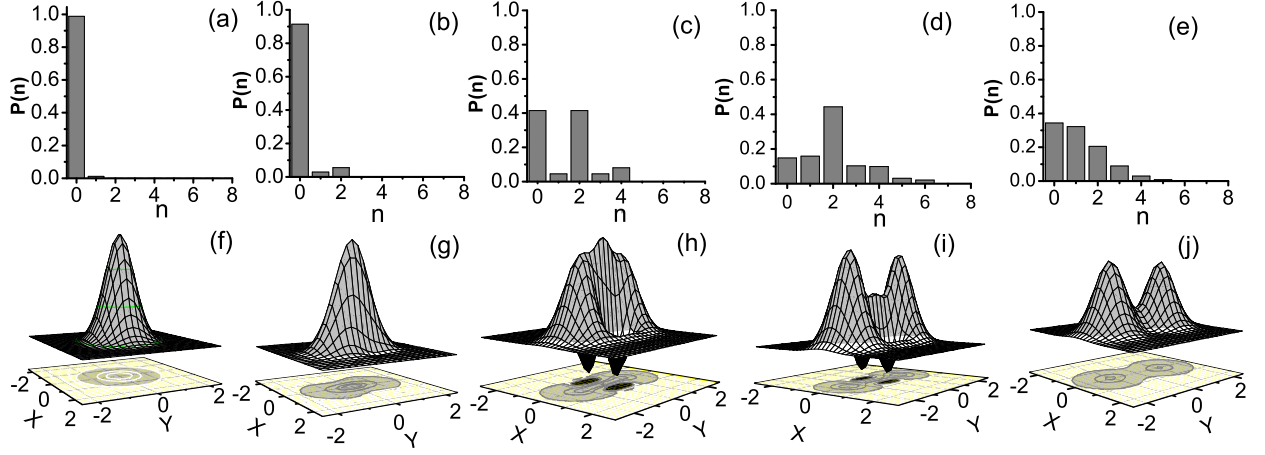


FIG. 4. The excitation number distributions for definite time-intervals (gray points noted in Figs. 3(a)):  $t = 2\tau - 2T$  (a);  $t = 2\tau - 1.8T$  (b);  $t = 2\tau - 0.4T$  (c);  $t = 2\tau$  (d);  $t = 2\tau + 0.6T$  (e). The Wigner functions corresponding these distributions for the same time-intervals. The black circles indicate the negative parts of the Wigner functions. The parameters are:  $\Delta/\gamma = -2$ ,  $\chi/\gamma = 5$ ,  $E\Omega/\gamma = 10$ ,  $\tau = 4\gamma^{-1}$ ,  $T = 0.5\gamma^{-1}$ .

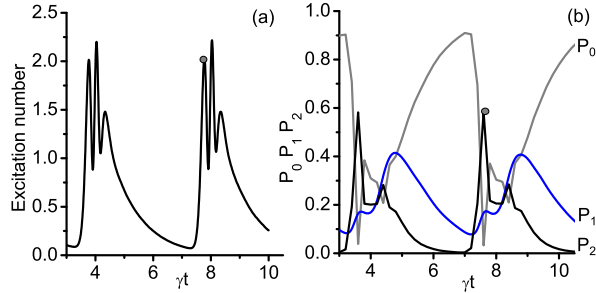


FIG. 5. Time evolution of averaged excitation number (a) and populations of Fock states (b). The parameters are:  $\Delta/\gamma = -10$ ,  $\chi/\gamma = 5$ ,  $E\Omega/\gamma = 10.3$ ,  $\tau = 4\gamma^{-1}$ ,  $T = 0.5\gamma^{-1}$ .

with phase modulation. Preparation of quantum states in PDAO has been stipulated by strong Kerr nonlinearity as well as by two-quanta resonant condition. We have studied the role of phase-localizing processes on production of oscillatory nonclassical states on the level of few excitation numbers. In this way, the production of the states approximately close to the superposition state  $|\Psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |2\rangle)$  and  $|2\rangle$  Fock state have been de-

scribed.

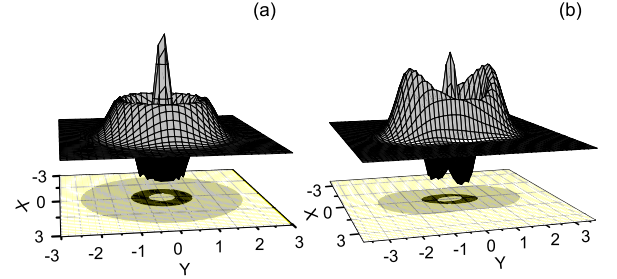


FIG. 6. The Wigner function for pure  $|2\rangle$  state (a). The Wigner function for  $t = 2\tau - 0.8T$  (b), for the parameters:  $\Delta/\gamma = -10$ ,  $\chi/\gamma = 5$ ,  $E\Omega/\gamma = 10.3$ ,  $T = 0.5\gamma^{-1}$ ,  $\tau = 4\gamma^{-1}$ . The ranges of negativity are indicated in the black.

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[1] M. A. Nielsen, I. L. Chuang, Quantum Computation and Quantum Information, Cambridge Univ. Press, (2000).

[2] C. K. Law, J. H. Eberly, Phys. Rev. Lett. **76**, 1055 (1996).

- [3] D. M. Meekhof, et al., Phys. Rev. Lett. **76**, 1796 (1996).
- [4] B. T. H. Varcoe, et al., Nature **403**, 743 (2000); P. Bertet, et al., Phys. Rev. Lett. **88**, 143601 (2002); E. Waks, E. Dimanti, Y. Yamamoto, N. J. Phys. **8**, 4 (2006).
- [5] M. Hofheinz, et al., Nature **454**, 310 (2008); Nature **459**, 546 (2009).
- [6] T. V. Gevorgyan, A. R. Shahinyan, and G. Yu. Kryuchkyan, Phys.Rev.A **85**, 053802 (2012).
- [7] B. Wielinga and G.J. Milburn, Phys. Rev. A **48**, 2494 (1993); Phys. Rev. A **49**, 5042 (1994).
- [8] F. DiFilippo, V. Natarajan, K.R. Boyce and D.E. Pritchard, Phys. Rev. Lett. **68**, 2859 (1992).
- [9] G. Yu. Kryuchkyan and K. V. Kheruntsyan, Opt. Comm. **120**, 132 (1996).
- [10] K.V. Kheruntsyan, D.S. Krahmer, G.Yu. Kryuchkyan, K.G. Petrossian, Opt. Comm. **139**, 157 (1997).
- [11] I. Fushman, et al. Science **320**, 769 (2000).
- [12] Q. A. Turchette, et al., Phys. Rev. Lett. **75**, 4710 (1995).
- [13] A. Imamoglu, et al., Phys. Rev. Lett. **79**, 1467 (1997); M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).
- [14] H. G. Craighead. Science, **290**, 1532 (2000).
- [15] K. L. Ekinci and M. L. Roukes. Rev. of Scientific Instruments, **76**, 061101 (2005).
- [16] Y. Makhlin, G. Schön, A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
- [17] D. I. Schuster, et al., Nature **445**, 515 (2007).
- [18] A. Fragner, et al., Science **322**, 1357 (2008).
- [19] O. Astafiev, et al., Nature **449**, 588 (2007).
- [20] M. Neeley, et al., Science **325**, 722 (2009).
- [21] O. Astafiev, et al., Science **327**, 840 (2010).
- [22] E. Hoskinson, et al., Phys. Rev. Lett. **102**, 097004 (2009).
- [23] J. Claudon, et al., Phys. Rev. B **78**, 184503 (2008).
- [24] R. Vijay, M. H. Devoret, and I. Siddiqi, Phys. Rev. A **80**, 111101 (2009).
- [25] S. Rebic, J. Twamley, and G. J. Milburn, Phys. Rev. Lett. **103**, 150503 (2009).
- [26] J. Q. You and F. Nori, Nature **474**, 585 (2011).
- [27] P. Kinsler and P. D. Drummond, Phys. Rev. A **43**, 6194 (1991); P. D. Drummond and P. Kinsler, Quantum Semiclass. Opt. **7**, 727 (1995); P. Kinsler and P. D. Drummond, Phys. Rev. A **52**, 783 (1995).
- [28] I. C. Percival, Quantum State Diffusion(Cambridge University Press, Cambridge), (2000).
- [29] H. H. Adamyan, S. B. Manvelyan and G. Yu. Kryuchkyan, Phys. Rev. A **63**, 022102 (2001).
- [30] G. Yu. Kryuchkyan and S. Manvelyan, Phys. Rev. Lett. **88**, 094101 (2002); H. H. Adamyan, S. B. Manvelyan and G. Yu. Kryuchkyan, Phys. Rev. E **64**, 046219 (2001).
- [31] T. V. Gevorgyan, A. R. Shahinyan, G. Yu. Kryuchkyan, Modern Optics and Photonics: Atoms and Structured Media. Eds: G. Kryuchkyan, G. Gurzadyan and A. Papoyan, World Scientific, 60-77, (2010).
- [32] T. V. Gevorgyan, A. R. Shahinyan, and G. Yu. Kryuchkyan, Phys. Rev. A **79**, 053828 (2009).
- [33] P. D. Drummond and D. F. Walls, J. Phys. A: Math. Gen **13**, 725 (1980).
- [34] L. Viola and S. Lloyd, Phys. Rev. A **58**, 2733 (1998); D. Vitali and P. Tombesi, ibid. **59**, 4178 (1999).
- [35] N. H. Adamyan, H. H. Adamyan, and G. Yu. Kryuchkyan, Phys. Rev. A **77**, 023820 (2008).